An introduction to induced-saturated graphs

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Following Axenovich and Csikós [1], we say that a graph G is H-induced-saturated if G does not contain H as an induced subgraph, but deleting any edge of G creates an induced copy of H and also adding any edge to G from its complement creates an induced copy of H. In other words, H-induced-saturated graphs are exactly the graphs in the intercesction of the class of all edge-minimal H-free graphs and the class of all edge-maximal H-free graphs.

It is non-trivial that an H-induced-saturated graph exists for a given H. Clearly, if H is a complete graph (on at least three vertices) or its complement, then no graph is H-induced-saturated. Also, Martin and Smith [1] observed that there is no H-induced-saturated graph for H chosen as a path on four vertices. It appears that P_4, K_k and kK_1 (where $k \geq 3$) are the only known choices of H such that no H-induced-saturated graph exists. On the other hand, V. Dvořák [5] constructed a P_k-induced-saturated graph for every k greater than five, and constructions for P_5 were found by Bonamy et al. [2] (see also [7, 4]). Behrens et al. [3] constructed H-induced-saturated graphs for H chosen as a star, odd cycle, matching. Axenovich and Csikós [1] characterised graphs H for which the Cartesian product of two complete graphs is H-induced-saturated.

In the talk, we discuss the known results on induced-saturated graphs and outline new constructions and open questions for possible investigation. Many of the present ideas come from discussing the topic with Pavel Dvořák, Tomáš Kaiser, Michal Opler, Théo Pierron and Aneta Šťastná.

References

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